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Monopole decay in the external electric field

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Abstract: The possibility of the magnetic monopole decay in the constant electric field is investigated and the exponential factor in the probability is obtained. Corrections due to Coulomb interaction are calculated. The relation between masses of particles for the process to exist is obtained.

Keywords: Nonperturbative Effects, Solitons Monopoles and Instantons.

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1. Introduction

Tunneling processes are very interesting nonperturbative phenomena. One can find the example of such process already in quantum mechanics, where it causes the energy splitting. There are tunneling processes in the field theory as well, for instance, pair production in external electromagnetic fields [1], decay of the false vacuum [4, 5].

In some spontaneously broken gauge theories there are magnetic monopole and dyon solutions. It is supposed that they can be produced in strong enough external electromagnetic fields. In the weak coupling regime their masses are huge, and sizes are of order $\sim 1/e^2M$. The probability of magnetic monopole pair production in constant magnetic field was calculated in the work of Affleck and Manton [3] using the instanton method. In the work of Bachas and Porrati [9] the rate of pair production of open bosonic and supersymmetric strings in a constant electric field was calculated exactly. In the work of Gorsky, Saraikin and Selivanov [7] stringy deformed probability of monopole and W-boson pair production was obtained quasiclassically. It is possible for particles like monopole, dyon or W-boson to decay nonperturbatively in external fields [7]. Monopole can also decay nonperturbatively in the external 2-form field [11].

In this paper we consider the process of magnetic monopole decay into electron and dyon, and W-boson decay into dyon and monopole using instanton method. Euclidean configuration corresponding to the monopole decay is represented on the figure 3. Exponential factor in the probability is given by the minimum of the electron, dyon and monopole total effective action (see (3.4)). This leads to (3.11) for the classical action. When monopole mass is equal to zero, one gets well-known result for exponential dependence of probability for pair production in external field. Dyon is not pointlike particle, so to apply this method for calculation one must imply that the size of dyon is much smaller then the size of electron-dyon loop. So, there is some condition imposed on the external field. The approximation used in this case is analogous to the thin wall approximation in the problem of the false vacuum decay. There is also condition of the dyon stability $M_d < M_m + m$, where m, M_d, M_m are masses of electron, dyon and monopole respectively. Contrary to spontaneous pair production, the process of the particle decay doesn't occur for arbitrary masses. It is shown that for the case when the relation (3.45) is fulfilled there are two negative eigenmodes, so, there is no decay at all. Coulomb corrections are taken into account similar to the work [2] in the limit $M_d \gg m$.

2. Spontaneous particle production

2.1 e^-e^+ pair production

One can obtain Schwingers [1] result for probability of e^-e^+ pair production summing the diagrams for vacuum amplitude similar to one represented on figure 1:

$$S_0 = \langle 0 \mid S \mid 0 \rangle = \det(i\widehat{\partial} - m) \cdot \det\left(1 - \frac{i}{i\widehat{\partial} - m}(-ie\widehat{A})\right). \tag{2.1}$$

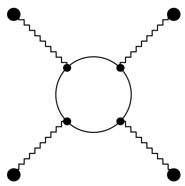

Figure 1: Diagrams for pair production process.

The probability connected with the amplitude is of the form

$$|S_0|^2 = \exp\left(-\int d^4x w(x)\right), \tag{2.2}$$

where $w(x)$ is the probability of pair production per unit time per unit volume.

$$2\ln S_0 = sp \ln\left(\left(\left(\widehat{P} - e\widehat{A}(x)\right)^2 - m^2\right)\frac{1}{P^2 - m^2}\right). \tag{2.3}$$

Using the representation

$$\ln\frac{a}{b} = \int_0^\infty \frac{ds}{s}\left(e^{is(b+i\varepsilon)} - e^{is(a+i\varepsilon)}\right), \tag{2.4}$$

one obtains

$$\Gamma = w(x) = -\frac{1}{(2\pi)^2}\int_0^\infty \frac{ds}{s^2}\left(eE\coth(eEs) - \frac{1}{s}\right)\mathrm{Re}(ie^{-is(m^2 - i\varepsilon)}). \tag{2.5}$$

So, the probability for the process of e^-e^+ pair production looks as follows

$$\Gamma = 2\frac{(eE)^2}{(2\pi)^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{\pi m^2}{eE}n\right). \tag{2.6}$$

2.2 Instanton method

The result obtained in the previous section is valid only for small coupling constant, since it doesn't take into account self interaction of electron loop (see figure 2). Authors of [3] have derived the expression for probability of monopole-antimonopole pair creation in external magnetic field using instanton method. As we know the rate of decay follows from the imaginary part of the ground state energy

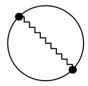

Figure 2: Photon exchange in the loop.

$$\Gamma' = 2\mathrm{Im}E_0\,, \tag{2.7}$$

where the energy E_0 is obtained from

$$e^{-E_0T} \approx \lim_{T\to\infty}\int \mathcal{D}x\mathrm{e}^{-S[x]}\,. \tag{2.8}$$

Using the WKB approximation one can do the integral and find the necessary probability per unit time per unit volume

$$\Gamma = 2\frac{\mathrm{Im}E_0}{V} = 2\mathrm{Im}\Delta\mathrm{e}^{-S_{cl}}\,, \tag{2.9}$$

where S_{cl} is the classical action calculated on the instanton solution, and Δ is one loop factor arising from the second variation of action. We should note that operator of the second variation must have one and only one negative eigenvalue. Affleck and Manton have calculated Δ and S_{cl} with the following result

$$\Gamma_M = \frac{(gB)^2}{(2\pi)^3}\mathrm{e}^{-\frac{\pi M^2}{gB}+\frac{g^2}{4}}\,, \tag{2.10}$$

where M, g are the mass and the charge of the monopole and B is the strength of the external magnetic field.

The exponential factor in the probability can be immediately obtained by minimizing the effective action

$$S_{\mathrm{eff}} = ML - gBQ\,, \tag{2.11}$$

where L is the length of classical monopole trajectory in the magnetic field, Q is the area restricted by this trajectory. Note that recently instanton approach has been used for calculation of the probability of the pair production in the nonhomogeneous fields [10] and in gravitational background [12].

3. Monopole decay

3.1 Exponential factor in probability

Now let us turn to the calculation of probability of the monopole decay in the external electric field. It was argued [7] that particles like monopole (W-boson) can decay in the external electric (magnetic) field into electron and dyon (dyon and antimonopole), the junction such as (DeM) naturally appears in the string theory. To find the probability we

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have to calculate the correction to Green's function in the presence of electron and dyon. Green's function of free heavy monopole in external electro-magnetic field in euclidean time can be written as:

$$G(T,0;0,0) = \int \mathcal{D}y_\mu \mathrm{e}^{-M_m \int \sqrt{\dot{y_\mu}^2}dt} \sim \mathrm{e}^{-M_m T}\,. \tag{3.1}$$

Taking into account one bounce correction we have

$$\begin{aligned} G(T,0;0,0)_{\text{bounce}} \sim \int \mathcal{D}x\mathcal{D}z \exp\Bigg(&-m\int\sqrt{\dot{x}^2}dt - M_d\int\sqrt{\dot{z}^2}dt - M_m(T-h) - \\ &- ie\int (A_\mu(x) + A_\mu^{ext}(x))dx_\mu + \\ &+ ie\int (A_\mu(z) + A_\mu^{ext}(z))dz_\mu - \int \frac{1}{4}F_{\mu\nu}^2 d^4x\Bigg)\,, \end{aligned} \tag{3.2}$$

where expression in the exponent is the well-known action for relativistic particles interacting with electromagnetic field. This correction is the first term of expansion of full Green's function $\sim e^{-(M_m+\delta M_m)T}$, and as we know, twice the imaginary part of the mass yields the probability of the decay. We consider the external field in Minkowski space as constant electric field aligned in the x_1 direction. The euclidean version of the field is of the form

$$A_\mu = \frac{i}{2}\left(Ex^1, -Ex^0, 0, 0\right). \tag{3.3}$$

Euclidean action for small coupling constant e^2 can be written as

$$S = ml + M_d L - eEQ - M_m h\,, \tag{3.4}$$

where l, L are the lengths of the trajectories and Q is the area of the region restricted by trajectories. We'll do path integral by steepest descent approximation. At first we find solution to the classical equation of motion. For the electron we have

$$m\frac{d}{dt}\frac{\dot{x}_\mu}{\sqrt{\dot{x}^2}} = -ieF_{\mu\nu}\dot{x}_\nu\,, \tag{3.5}$$

while for the dyon

$$\frac{d}{dt}\frac{\dot{z}_\mu}{\sqrt{\dot{z}^2}} = ieF_{\mu\nu}\dot{z}_\nu\,, \tag{3.6}$$

where we neglect interaction between particles, i.e. Coulomb effects. Solutions to these equations are arcs of circles with radii $r = \frac{m}{eE}$, $R = \frac{M_d}{eE}$. Introducing the angles that define the lengths of the arcs θ_1, θ_2 for electron and dyon respectively (figure 4) we have the action:

$$\begin{aligned} S_{cl} &= 2mr\theta_1 + 2M_d R\theta_2 - 2M_m r\sin\theta_1 - eE\theta_1 r^2 + \\ &\quad + \frac{1}{2}eEr^2\sin 2\theta_1 - eE\theta_2 R^2 + \frac{1}{2}eER^2\sin 2\theta_2\,. \end{aligned} \tag{3.7}$$

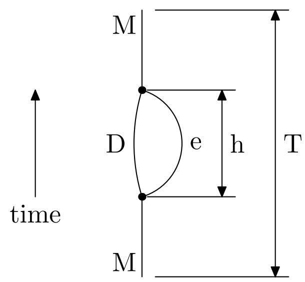

Figure 3: Classical trajectories of particles in euclidean time.

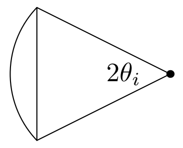

Figure 4: Square of segment.

In order to find θ_1, θ_2, r, R we minimize action taking into account that

$$r \sin\theta_1 = R \sin\theta_2 \,. \tag{3.8}$$

Adding to the action term $(r\sin\theta_1 - R\sin\theta_2)$ with the Lagrange multiplier λ one gets

$$\begin{cases} 2mr - 2M_m r\cos\theta_1 + \lambda r\cos\theta_1 - eEr^2 + eEr^2\cos 2\theta_1 = 0\,, \\ 2M_d R - \lambda R\cos\theta_2 - eER^2 + eER^2\cos 2\theta_2 = 0\,, \\ r\sin\theta_1 - R\sin\theta_2 = 0\,, \\ 2m\theta_1 - 2M_m\sin\theta_1 + \lambda\sin\theta_1 - 2eE\theta_1 r + eEr\sin 2\theta_1 = 0\,, \\ 2M_d\theta_1 - \lambda\sin\theta_2 - 2eE\theta_2 R + eER\sin 2\theta_2 = 0\,. \end{cases} \tag{3.9}$$

Solution to this system is

$$\begin{cases} r = \dfrac{m}{eE}\,, \\ R = \dfrac{M_d}{eE}\,, \\ \theta_1 = \arccos\dfrac{M_m^2 + m^2 - M_d^2}{2mM_m}\,, \\ \theta_2 = \arcsin\left(\dfrac{m}{M_d}\sin\theta_1\right)\,, \end{cases} \tag{3.10}$$

which amounts to the action

$$S_{cl} = \frac{m^2}{eE}\arccos\frac{M_m^2 + m^2 - M_d^2}{2mM_m} + \frac{M_d^2}{eE}\arccos\frac{M_m^2 - m^2 + M_d^2}{2M_dM_m} - \\ - \frac{mM_m}{eE}\sqrt{1 - \left(\frac{M_m^2 + m^2 - M_d^2}{2mM_m}\right)^2}\,. \tag{3.11}$$

This result, of course, could be obtained in the way similar to that described in [8] for the problem of induced false vacuum decay. Classical trajectories of electron, dyon and monopole are determined from the equilibrium condition in vertex via mechanical analogy (m, M_d are surface tensions and M_m external force)

$$\begin{cases} m\sin\theta_1 = M_d\sin\theta_2\,, \\ M_m - m\cos\theta_1 - M_d\cos\theta_1 = 0\,. \end{cases} \tag{3.12}$$

It follows from these equations that if M_m equals to zero then m and M_d will be equal to each other, i.e. creation of particles with different masses is accompanied by decay of some massive particle. Also we should note that such action for dyon and monopole implies that these particles have no size. However, since these particles are not pointlike the approximation used works only for small enough external field. The size of dyon ($\sim 1/e^2 M_d$) should be much smaller then the size of electron-dyon loop ($\sim m/eE$), so the field should obey the condition

$$E \ll e m M_d \,. \tag{3.13}$$

Also we should make some comment on the limit $M_m = 0$, which implies the condition $m = M_d$. Intuitively it seems to coincide with the result of circular symmetrical case of pair production. The action for this case, as one can see from (3.11), becomes

$$\frac{\pi m^2}{eE} \,. \tag{3.14}$$

which is the same as in the circular symmetric case.

3.2 W-boson decay

Let us make a short digression on the W-boson decay in the magnetic field. It was argued that W-boson can decay nonperturbatively in external magnetic field into monopole and dyon as magnetic monopole in external electric field [7] into electron and dyon. However, since strong enough magnetic fields occur in cosmology more often then strong electric fields the formula for W-boson decay could be even more useful. The problem of W-boson decay is identical to one of monopole decay since there is dual symmetry between electric and magnetic fields. So, the result for the probability of W-boson decay reads as

$$\Gamma_w = A \mathrm{e}^{\frac{M_m^2}{gB} \arccos \frac{M_w^2 + M_m^2 - M_d^2}{2 M_w M_m} + \frac{M_d^2}{gB} \arccos \frac{M_w^2 - M_m^2 + M_d^2}{2 M_d M_w} - \frac{M_w M_m}{gB} \sqrt{1 - \left(\frac{M_w^2 + M_m^2 - M_d^2}{2 M_w M_m} \right)^2}} \,, \tag{3.15}$$

where M_w, M_m, M_d are masses of W-boson, monopole and dyon respectively, B is external magnetic field and g is magnetic charge of monopole.

3.3 Determinant

Now we return to the monopole decay. It was mentioned before that the classical trajectories of electron and dyon are the arcs of the circles. For the electron we have

$$\begin{cases} x_0^{cl} &= r \sin(2\theta_1 t - \theta_1) \,, \\ x_1^{cl} &= -r \cos(2\theta_1 t - \theta_1) + r \cos\theta_1 \,. \end{cases} \tag{3.16}$$

Operator of the second variation looks as follows

$$-\frac{m}{\sqrt{\dot{x}_{cl}^2}} \delta_{\mu\nu} \frac{d^2}{dt^2} + \frac{m}{(\dot{x}_{cl}^2)^{3/2}} \dot{x}_\mu^{cl} \dot{x}_\nu^{cl} \frac{d^2}{dt^2} + \frac{m}{(\dot{x}_{cl}^2)^{3/2}} (\ddot{x}_\mu^{cl} \dot{x}_\nu^{cl} + \dot{x}_\mu^{cl} \ddot{x}_\nu^{cl}) \frac{d^2}{dt^2} - ieF_{\mu\nu} \frac{d}{dt} \,, \tag{3.17}$$

where

$$\sqrt{\dot{x}_{cl}^2} = 2\theta_1 r = 2\theta_1 \frac{m}{eE} \,. \tag{3.18}$$

Let us define

$$a = 2\theta_1 t - \theta_1 \,, \tag{3.19}$$

then we get the equations for eigenfunctions and eigenvalues

$$\left\{ \begin{aligned} -\frac{1}{4\theta_1} f_0'' + \frac{1}{4\theta_1} \cos 2a f_0'' - \sin 2a f_0' + \frac{1}{4\theta_1} \sin 2a f_1'' + \cos 2a f_1' - f_1' &= \frac{\lambda}{eE} f_0 \,, \\ \frac{1}{4\theta_1} \sin 2a f_0'' + \cos 2a f_0' + f_0' - \frac{1}{4\theta_1} f_1'' - \frac{1}{4\theta_1} \cos 2a f_1'' + \sin 2a f_1' &= \frac{\lambda}{eE} f_1 \,, \\ -\frac{1}{2\theta_1} f_2'' &= \frac{\lambda}{eE} f_2 \,, \\ -\frac{1}{2\theta_1} f_3'' &= \frac{\lambda}{eE} f_3 \,. \end{aligned} \right. \tag{3.20}$$

To solve these equations denote

$$\begin{aligned} F &= f_0 + i f_1 \,, \\ \bar{F} &= f_0 - i f_1 \,, \end{aligned} \tag{3.21}$$

and multiplying the second equation in (3.20) by i and adding to the first equation we get

$$-\frac{1}{4\theta_1} F'' + \frac{1}{4\theta_1} \mathrm{e}^{2ia} \bar{F}'' + i\mathrm{e}^{2ia} \bar{F}' + i\bar{F}' = \frac{\lambda}{eE} F \,. \tag{3.22}$$

One can obtain complex conjugated equation by multiplying the second equation by i and subtracting it from the first one. The shape of the equation tells us the form of the solution

$$\left\{ \begin{aligned} F &= \mathrm{e}^{-ia} g \,, \\ g &= g_1 + i g_2 \,. \end{aligned} \right. \tag{3.23}$$

Upon this substitution we have

$$\left\{ \begin{aligned} -\frac{1}{2\theta_1} g_2'' - 2\theta_1 g_2 &= \frac{\lambda}{eE} g_2 \,, \\ \lambda g_1 &= 0 \,, \end{aligned} \right. \tag{3.24}$$

and

$$\left\{ \begin{aligned} f_0^e(t) &= (A_1 \cos\omega_1 t + B_1 \sin\omega_1 t) \sin(2\theta_1 t - \theta_1) \,, \\ f_1^e(t) &= -(A_1 \cos\omega_1 t + B_1 \sin\omega_1 t) \cos(2\theta_1 t - \theta_1) \,, \end{aligned} \right. \tag{3.25}$$

$$\lambda = eE \left(\frac{\omega_1^2}{2\theta_1} - 2\theta_1 \right) . \tag{3.26}$$

Similar manipulations for the dyon yield

$$\left\{ \begin{aligned} f_0^d(t) &= (A_2 \cos\omega_2 t + B_2 \sin\omega_2 t) \sin(2\theta_2 t - \theta_2) \,, \\ f_1^d(t) &= (A_2 \cos\omega_2 t + B_2 \sin\omega_2 t) \cos(2\theta_2 t - \theta_2) \,. \end{aligned} \right. \tag{3.27}$$

$$\lambda = eE \left(\frac{\omega_2^2}{2\theta_2} - 2\theta_2 \right) . \tag{3.28}$$

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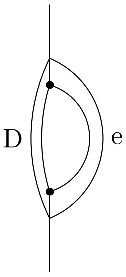

Figure 5: Perturbation corresponding to the increasing of radii of trajectories.

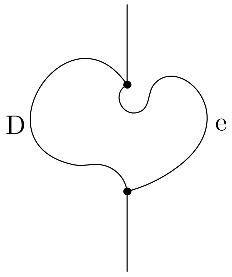

Figure 6: The allowed perturbed configuration.

There are zero eigenmodes for every particle corresponding to the translations. The mode corresponding to the translation in x_1 direction is

$$\begin{cases} f_0 = 0\,, \\ f_1 = 1\,. \end{cases} \tag{3.29}$$

However, these perturbations for electron and dyon are not independent. Since worldline of monopole is straight with $x_1 = 0$ (see figure 6), the unification of distorted trajectories should be closed. The resulting electron-dyon loop should have vertexes with at $x_1 = 0$. To satisfy the last condition one can require

$$f_1(0) = f_1(1)\,, \tag{3.30}$$

and using zero mode translate perturbed trajectory to $x_1 = 0$. To satisfy the former condition one must require

$$f_0^e(1) - f_0^e(0) = f_0^d(1) - f_0^d(0)\,. \tag{3.31}$$

Let us find solutions consistent with these restrictions. For the solution with factor $\cos \omega t$ we get

$$f_1(0) = f_1(1) = A \cos\theta = A \cos\omega \cos\theta\,, \tag{3.32}$$

so, $\omega = 2\pi n$. For the solution with factor $\sin \omega t$ we get

$$f_1(0) = f_1(1) = 0 = B \sin\omega \cos\theta\,, \tag{3.33}$$

that is $\omega = \pi n$. Finally we have

$$\begin{cases} f_0^{e,d}(t) = B_{1,2} \sin \pi t \sin(2\theta_{1,2} t - \theta_{1,2})\,, \\ f_1^{e,d}(t) = \mp B_{1,2} \sin \pi t \cos(2\theta_{1,2} t - \theta_{1,2})\,, \end{cases} \tag{3.34}$$

with eigenvalues

$$\lambda_n^{e,d} = eE \left(\frac{(\pi n)^2}{2\theta_i} - 2\theta_i \right), \tag{3.35}$$

and

$$\begin{cases} f_0^{e,d}(t) = A_{1,2} \cos 2\pi n t \sin(2\theta_{1,2} t - \theta_{1,2})\,, \\ f_1^{e,d}(t) = \mp A_{1,2} \cos 2\pi n t \cos(2\theta_{1,2} t - \theta_{1,2})\,, \end{cases} \tag{3.36}$$

with eigenvalues

$$\lambda_n^{e,d} = eE\left(\frac{(2\pi n)^2}{2\theta_i} - 2\theta_i\right). \tag{3.37}$$

It is obvious that first set of solutions already fulfills (3.31), so, for this set we have independent perturbations for the electron and for the dyon. The second set do not fulfills (3.31) for arbitrary A_1 and A_2. They must satisfy the condition

$$A_1 \sin\theta_1 = A_2 \sin\theta_2\,, \tag{3.38}$$

thus

$$\frac{A_1}{A_2} = \frac{m}{M_d}\,. \tag{3.39}$$

It is quite difficult to find full quantitative expression for the determinant, but we can investigate it qualitatively. We are interested in the presence of negative eigenmodes. It is easy to see from (3.37) that if $n = 0$, some eigenvalue becomes negative. Corresponding eigenfunctions are proportional to the classical solutions

$$\begin{cases} f_0^e = am\sin(2\theta_1 t - \theta_1)\,, \\ f_1^e = -am\cos(2\theta_1 t - \theta_1)\,, \end{cases} \tag{3.40}$$

$$\begin{cases} f_0^d = aM_d\sin(2\theta_2 t - \theta_2), \\ f_1^d = aM_d\cos(2\theta_2 t - \theta_2), \end{cases} \tag{3.41}$$

and they are responsible for simultaneous increasing of the radii of the trajectories (see figure 5), where a is the corresponding parameter. Similar negative eigenmode exists in the case of spontaneous pair production (see for example [2]). One can calculate the action on perturbed configuration, expand it in power series by small deviation from classical radius and find that the quadratic correction is negative. Hence this perturbation is connected to the negative eigenmode. Indeed, correction to the action proportional to a^2 reads as

$$S_{(2)} = \frac{m^2a^2}{2}(-2\theta_1 + \sin 2\theta_1) + \frac{M_d^2a^2}{2}(-2\theta_2 + \sin 2\theta_2) < 0\,, \tag{3.42}$$

since $x > \sin x$.

We can also consider the first set of solutions (3.34). The solution corresponding to the particle with smaller mass, i.e. to the electron, for $n = 1$ is

$$\begin{cases} f_0^e = B_1\sin\pi t\sin(2\theta_1 t - \theta_1)\,, \\ f_1^e = -B_1\sin\pi t\cos(2\theta_1 t - \theta_1)\,, \end{cases} \tag{3.43}$$

and

$$\lambda^e = eE\left(\frac{\pi^2}{2\theta_1} - 2\theta_1\right). \tag{3.44}$$

It is obvious that $\lambda < 0$ when $\theta_1 > \frac{\pi}{2}$ (see figure 7) that is

$$M_d^2 > M_m^2 + m^2\,, \tag{3.45}$$

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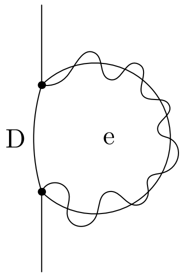

Figure 7: Perturbation of the electron's trajectory.

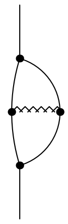

Figure 8: Coulomb interaction.

and the condition of dyon stability is

$$M_d^2 < (M_m + m)^2 = M_m^2 + m^2 + 2mM_d \,. \tag{3.46}$$

One can also calculate correction to the action

$$S_{(2)} = \left(-\frac{1}{2}\theta_1 + \frac{\pi^2}{8\theta_1} \right) B_1^2 \,. \tag{3.47}$$

It is negative provided that condition on masses (3.45) is fulfilled. Since the presence of the negative eigenmode connected to the changing of configuration's size do not depend on this condition, we have two negative eigemodes in this case. Hence, the leading factor of the probability in the semi-classical approximation becomes zero. In this case the action on the euclidean configuration provides the nonperturbative renormalization of monopole's mass.

3.4 Coulomb corrections

Now we'll calculate the quantum corrections due to photon exchange between electron and dyon (see figure 8). We know that

$$\int \mathcal{D}A A_\mu(x) A_\nu(x^{'}) \mathrm{e}^{-\frac{1}{4}\int F_{\mu\nu}^2 d^4x} = \frac{1}{4\pi^2} \frac{g_{\mu\nu}}{(x-x^{'})^2} \,, \tag{3.48}$$

and therefore using the relation above we obtain the contribution to the path integral from electromagnetic field

$$\begin{aligned}
&\int \mathcal{D}A \mathrm{e}^{-\frac{1}{4}\int F_{\mu\nu}^2 d^4x + ie\oint A_\mu(x)dx_\mu} = \\
&\quad = \int \mathcal{D}A \mathrm{e}^{-\frac{1}{4}\int F_{\mu\nu}^2 d^4x} \left(1 + ie \oint A_\mu(x)dx_\mu + \frac{1}{2!}(ie)^2 \oint A_\mu(x)dx_\mu A_\nu(x^{'})dx_\nu^{'} + \cdots \right) \\
&\quad = \left(1 - \frac{e^2}{8\pi^2} \oint\oint \frac{g_{\mu\nu}}{(x-x^{'})^2} dx_\mu dx_\nu^{'} + \cdots \right) \\
&\quad = \mathrm{e}^{-\frac{e^2}{8\pi^2}\oint\oint \frac{g_{\mu\nu}}{(x-x^{'})^2} dx_\mu dx_\nu^{'}} \,.
\end{aligned} \tag{3.49}$$

We have to do the integral over the path which consists of trajectories of particles, schematically

$$\oint\oint = \int_{\text{electron}}\int_{\text{electron}} + \int_{\text{dyon}}\int_{\text{dyon}} - 2\int_{\text{electron}}\int_{\text{dyon}} \,. \tag{3.50}$$

It is difficult to calculate this integral in general case hence we'll do it for the case $M_d \gg m$, so we can consider trajectory of electron as semi circle and trajectory of dyon as a straight line. Using the dimensional regularization we get

$$\int_{\text{electron}} \int_{\text{electron}} = \frac{\pi^2}{2} \int_0^1 \frac{\cos \pi(t-\tau)}{1 - \cos \pi(t-\tau)} dt d\tau = -\frac{\pi^2}{2} . \tag{3.51}$$

Integration for dyon contribution gives

$$\int_{\text{dyon}} \int_{\text{dyon}} = 0 , \tag{3.52}$$

and for electron-dyon part

$$\int_{-1}^{1} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dt d\tau \frac{\cos t}{(\tau - \sin t)^2 + \cos^2 t} = \frac{\pi}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dt = \frac{\pi^2}{2} . \tag{3.53}$$

Finally, collecting all together we find for the case $M_d^2 < M_m^2 + m^2$ the probability of monopole decay is of the form

$$\begin{aligned} \Gamma \sim \exp\Bigg(& \frac{m^2}{eE} \arccos \frac{M_m^2 + m^2 - M_d^2}{2mM_m} + \frac{M_d^2}{eE} \arccos \frac{M_m^2 - m^2 + M_d^2}{2M_d M_m} - \\ & - \frac{mM_m}{eE} \sqrt{1 - \left(\frac{M_m^2 + m^2 - M_d^2}{2mM_m} \right)^2} + \frac{3e^2}{16} \Bigg) , \end{aligned} \tag{3.54}$$

while for the case $M_d^2 > M_m^2 + m^2$ similar exponential factor corresponds to the monoples mass renormalization.

$$\delta m = A \mathrm{e}^{-S_{cl} + S_{\text{int}}} . \tag{3.55}$$

For the case $M_d^2 = M_m^2 + m^2$, i.e. for BPS case the negative eigenmode (3.43) becomes zero eigenmode and the probability for the decay reads as

$$\Gamma \sim \exp\left(\frac{\pi m^2}{2eE} + \frac{(M_m + m)^2}{eE} \arccos \left(\frac{1}{2} \left(1 + \frac{m^2}{M_m^2} \right)^{-\frac{1}{2}} \right) - \frac{mM_m}{eE} + \frac{3e^2}{16} \right) . \tag{3.56}$$

4. Conclusion

In this paper the process of monopole decay in external electric field was investigated. The probability of monopole decay and W-boson decay up to the exponential accuracy has been found. It was shown that the result obtained is in agreement with one for the probability of pair production. The Coulomb corrections were calculated for the process of monopole decay in external electric field. The determinant was investigated qualitatively, calculation of full expression for the determinant will be made elsewhere. It was found that the leading semi-classical term of the monopole decay probability vanishes if the relation between masses (3.45) is fulfilled. But this result doesn't mean that there is no monopole decay. For example the stringy corrections to the probability of the decay can be not

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equal to zero. It deserves deeper investigation. It is obvious that the probability of such processes is exponentially suppressed, but they could have some applications in cosmology. The natural generalization of the problem would involve the temperature effects and the stringy corrections.

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